

Design of Planar Circuit Structures with an Efficient Magneto-Static Field Solver

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Abstract

We introduce a highly efficient magneto-static field solver for the design of low frequency lossless planar circuit elements with arbitrary geometry. The field solver is based on a finite difference formulation of a scalar magnetic potential, using potential partitioning surfaces (PPS). We present numerical results at the examples of various planar circuit elements.

Introduction

For the design of planar circuit elements the given structures are mostly extracted to a lumped element circuit model. In many applications, the structures are considered in a low frequency range in which the lumped elements of the model are nearly independent of the frequency. This given, the electromagnetic properties of the planar circuit elements can be described by considering the electro- and magneto-static fields in the structures. The given circuit elements may have a complicated geometry. With that a flexible CAD-tool for the efficient simulation of the static fields in lossless planar circuit structures with arbitrary geometry is required.

The demand for a method of simulating the field in structures with arbitrary geometry leads to a space discretizing method like the Finite-Difference method [1,2]. The structure of the circuit elements is discretized according to Yee's scheme [2]. For the simulation of the electro-static field Maxwell's equations are reduced to a Poisson-equation of the electro-static potential. This reduction of considering only a scalar potential instead of the three components of the electric field leads to considerable savings in computation time and storage.

While the electro-static field can be calculated in a fast way by means of a scalar potential the calculation of the magnetic field can not be performed directly in such a simple way. This is because, in contrast to the electric field, the contour integral of the magnetic field does not disappear if a conductor is enclosed. Thus in general, this would mean to calculate the field in terms of three components which makes the computational effort increase. The law of Biot-Savart can not be used for the calculation because the current distribution is unknown.

The PPS-Finite-Difference field solver

In this contribution we present a PPS-Finite-Difference (PPS-FD) field solver for the efficient simulation of the magneto-static field in arbitrary lossless planar circuit structures. Compared with a Finite Difference fullwave analysis the simulation of the magnetic field with the PPS-FD solver requires less than 5 % of the CPU-time and 33 % in memory. The solver is also applicable for a hybrid dynamic-static Finite-Difference method for efficient field simulation at higher frequencies, as presented in [3]. The PPS-FD-solver is based on the introduction of potential partitioning surfaces (PPS) into the structure, connecting each conductor in the structure with the outer boundary in a way that each integration path around the conducting material crosses this potential partitioning surface. Fulfilling this requirement the choice of the exact position of the PPS is arbitrary. Assuming the case of a lossless three-dimensional structure, the consideration of the field is reduced to the spatial region around the conducting material. This region is cut by the PPS so that the resulting subregion is bordered by two more surfaces which are both sides of the partitioning surface. In this new defined domain the magnetic field is irrotational and

hence it can be described by a scalar magnetic potential M , in analogy to the electro-static case:

$$\oint_C H ds = 0 \Rightarrow H = -\nabla M \quad (1)$$

For the description of the potential in the subregions, the divergence free magnetic field yields a simple differential equation which leads to a fast numerical algorithm for the calculation of the potential M .

$$\operatorname{div}(\mu \nabla M) = 0 \quad (2)$$

The boundary conditions of the field along electric and magnetic walls are defined in a dual way to the electro-static case. The integration of the field around a conductor from one side of the PPS to its other side yields a step of the potential when passing this PPS.

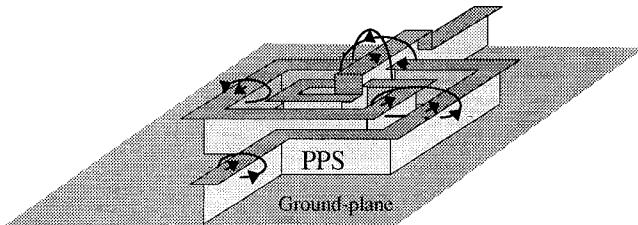


Fig.1: Example of incorporation of a potential partitioning surface (PPS) into a spiral structure.

Due to Ampere's law, the difference between the potential on one side and the other side of the partitioning surface is equivalent to the current in the conductor [4]. Thus, the integration of the magnetic field from a point P on one side of the partitioning surface to the point P^+ on the other side yields the difference of the potentials M at these points (Fig 1). This means that there is a step in potential which has the value of the current in the conductor.

$$\int_{P^-}^{P^+} H \cdot ds = I \Rightarrow M(P^+) - M(P^-) = I \quad (3)$$

Fig. 1 shows an example of incorporation of a potential partitioning surface (PPS) into a spiral structure. The PPS connects the conductor to the ground plane completely. Crossing PPS' are possible. a,b: The integration of H around the current I leads to a potential step of the value I . c: The Integration around two currents, passing the integration path in the same direction leads to two potential steps of the value I . e: The integration around two currents I , passing the

integration path in opposite direction leads to two potential steps of the values I and $-I$. With that the Inegration around a conductor is allways put in respect in the right way.

While the potential is non-continuous at the partitioning surface, all the derivatives have to be continuous. This results from the requirement that the magnetic field distribution is independent of the choice of the local position of the potential partitioning surface at the conductor.

$$H(P^+) = H(P^-) \Rightarrow \left. \frac{\partial^{l+m+n} M}{\partial x^l \partial y^m \partial z^n} \right|_{P^+} = \left. \frac{\partial^{l+m+n} M}{\partial x^l \partial y^m \partial z^n} \right|_{P^-} \quad (4)$$

(for $l,m,n>0$). Those properties lead to the unique description of the magnetic potential. The calculation of the potential and the magnetic field can now be performed in a fast numerical way. The differential equation (2) of the potential is realized in the Finite-Differencemethod in a cartesian mesh of elementary cells. The potentials M are defined on the hyphens of the mesh cells. Applying equation (2) in Finite-Difference form of a linear equation to each mesh cell of the given structure yields a solvable system of equations. Additionally potentials at the boundaries of the given structure are extracted by the boundary conditions: While magnetic walls are equipotential surfaces with given potential M , the relations between neighboring potentials on electric walls are given by symmetrical characteristics.

Also the steps in potential are realized as a source vector in the equation system. The system is then analogous to the Finite-Difference description system for the electro-static field. The solution of the magneto-static system of equations yields a vector in which all the potentials M of the structures cells are included. After that the magneto-static field can be determined by the Finite-Difference quotient of the magnetic potential distribution according to eqn. (1).

Numerical results

In the following we present numerical results of the PPS-FD solver. First we consider the magneto-static field in a coplanar spiral inductor on silicium substrate. As in the example of Fig.1, the PPS connects the spiral conductor vertically to the ground plane. The conductor route leading to the center of the spiral is crossed by the spiral in form of two air-

bridges. By following the spiral route the potential partitioning surface crosses itself below the air-bridges. In Fig. 2. we see the spiral structure with its discretization steps. The discretization in the inner space is not shown in this figure. In Fig.3 we see the H_x component in the spiral plane, which is obtained numerically by the PPS-FD solver.

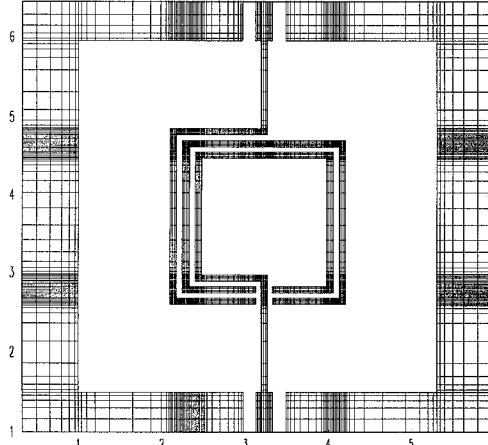


Fig. 2.: coplanar spiral inductivity with two air bridges, presentation in the spiral plane

The structure is discretized into half a million mesh cells. This means that the computation of the magnetic field with a FDFD full-wave analysis requires about 40 hours. In contrast to the full wave analysis the PPS-FD solver requires only about 1 hour for the magneto-static field of the spiral structure. This is the same low computational effort as for the analysis of the electro-static field. So, by using the PPS-FD solver for extracting the low frequency lumped element model of the spiral structure we have a reduction in computation time to 2.5%. With higher cell numbers there would even be a higher reduction. The required storage is reduced to 33%.

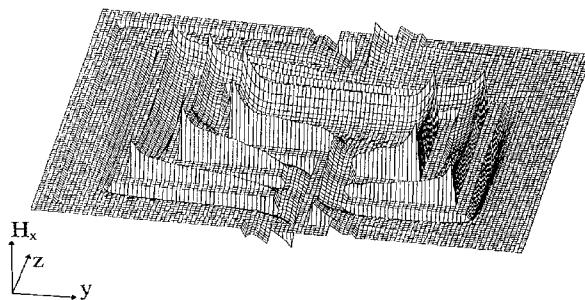


Fig. 3.: Field H_x of the spiral inductor in the spiral plane.

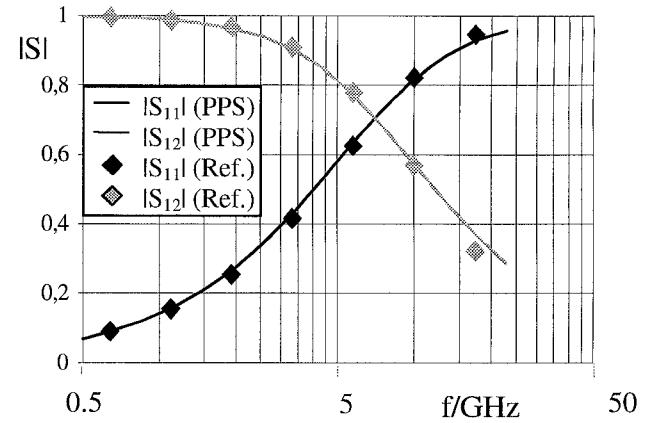


Fig. 4: Absolute values of S_{11} and S_{12} , calculated by the PPS-method, accurate reference solutions by Finite-Difference full-wave analysis

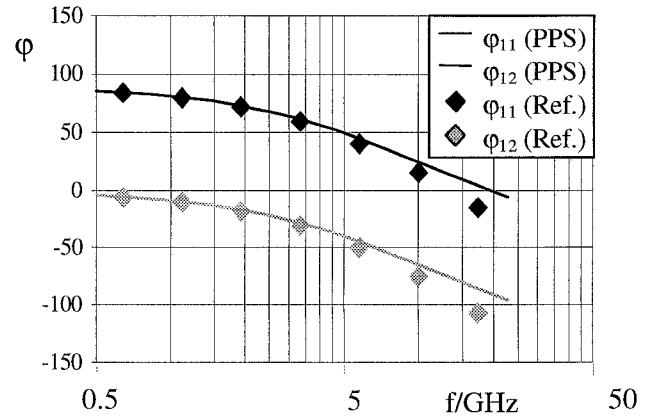


Fig. 5: Phases ϕ of S_{11} and S_{12} , calculated by the PPS-method, accurate reference solutions by Finite-Difference full-wave analysis

The inductance of the spiral structure is calculated easily by integrating the normal magnetic field along the PPS. After calculating also the capacity of the structure by the electromagnetic field, an equivalent model of the structure can be derived for the wavelength being ten times higher than the structure size. From the equivalent circuit model we derive the S-parameters which are shown in Fig. 4 and Fig. 5. They are compared to the S-parameters which were calculated by the FDFD full-wave analysis with 40 times higher computation time. we see a very good agreement of the s-parameters in the frequency range up to 5 GHz in phase and absolute value.

As a further example we consider a microstrip bend ($Z_c=50\Omega$) with edge compensation, as can be seen in Fig. 6. The PPS is again connecting the microstrip conductor to the ground plane.

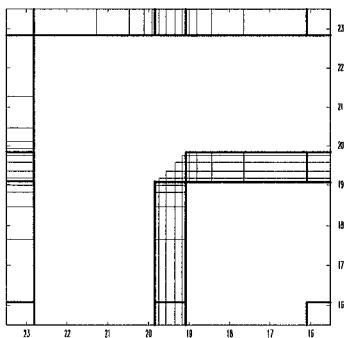


Fig. 6: Microstrip bend with edge compensation, presentation in the microstrip plane.
(sizes in $100\mu\text{m}$)

While the distance of the microstrip line to the ground plane is $100\mu\text{m}$, the distance of the line to the upper boundary of the box is $500\mu\text{m}$. Fig. 7 shows the calculated magneto-static field of the structure in the microstrip plane.

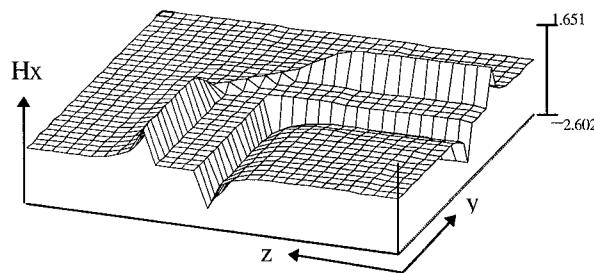


Fig. 7.: Field H_x of the microstrip bend in the microstrip plane.

For further verification of our results, we calculated the magneto-static field also for a triplate bend with edge compensation, which is identical to the microstrip bend of Fig. 6 but with symmetry to the x-axis. Using this symmetry the reference field has been calculated referring to a method of same computation effort, using also a scalar potential which has been presented in ref. [5]. without using PPS, this method is only restricted to structures of symmetry to the metallization plane like the triplate. This field is compared with the field, which was calculated by the

PPS-FD solver. The relative Difference of the results was in the range of 10^{-6} %. The method of ref. [5] has the same efficiency, but is restricted to symmetrical structures. Using the method of ref. [5] for the unsymmetric microstrip bend by approximation of the symmetric structure would lead to an unacceptable field deviation of 10%. Using the PPS-FD method this approximation is not necessary.

Conclusion

We present a highly efficient PPS-Finite-Difference solver for the fast calculation of the magneto-static field. The PPS-method leads to the calculation of a well defined scalar magnetic potential. It is applicable without restriction to all types of lossless structures. With that the numerical effort for the Finite-Difference calculation of the magnetic field is as low as in the electro-static case. Compared to the Finite Difference Method in frequency domain the effort on CPU-time for the magnetic field simulation is reduced to less than 5%. The storage requirement decreases to 1/3. The PPS method is applicable without restriction to all types of lossless structures. It can be used for calculating Inductances and for the application in a hybrid dynamic-static finite difference method.

References

- [1] S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antenna Propagat.*, Vol.14, pp. 302-307, May 1966.
- [2] T. Weiland, "On the numerical solution of Maxwellian eigenvalue problems in three dimensions," *Particle Accelerators*, Vol.17, pp. 227-242, 1985.
- [3] S. Lindenmeier, P. Russer, W. Heinrich, "Hybrid dynamic-static Finite-Difference approach for MMIC design," *1996 Int. Microwave Symposium Digest*, Vol. 1, pp. 197-200
- [4] Simonyi, „Theoretische Elektrotechnik“ Hochschul-bücher der Physik, Vol. 20, VEB Deutscher Verlag der Wissenschaft, Berlin 1979
- [5] M. Abdo-Tuko, M. Naghed, I. Wolff, "Novel 18/36 GHz (M)MIC GaAs FET frequency doublers in CPW-techniques under the consideration of the effects of coplanar discontinuities", *IEEE Trans. Microw. Theory Tech.*, Vol 41, No. 8, pp. 1307-1315, Aug. 1993